Tanta University
Faculty of engineering
Electrical Power and Machines engineering department
Energy Conversion Course

Sheet (4) …… 3-Phase Induction Motor

1) A 3 phase, 4 pole, 50 Hz, star connected induction motor running on full load develops a useful torque of 300 N.m. The rotor e.m.f. is completing 120 cycles per minute. If torque lost in friction is 50 N.m, calculate:
   a) Slip
   b) Net output power
   c) Rotor copper loss per phase
   d) Rotor efficiency
   e) Rotor resistance per phase if rotor current is 60 A in running condition

2) A 25 kW, 4 pole, 3 phase, 50 Hz induction motor is running at 1410 r.p.m., supplying full load. The mechanical losses are 850 W and stator losses are 1.7 times rotor copper losses on full load. Calculate:
   a) Gross mechanical power developed
   b) Rotor copper losses
   c) The value of rotor resistance per phase if rotor current on full load per phase is 65 A
   d) The full load efficiency

3) While delivering an useful power of 24 kW to the full load, a 3 phase, 50 Hz, 8 pole induction motor draws a line current of 57 A. It runs at a speed of 720 r.p.m. and is connected to 415 V supply. The p.f. of the motor is observed to be 0.707 lagging. Stator resistance per phase is 0.1 Ω. Mechanical losses are 1000 W. Calculate:
   a) Shaft torque
   b) Gross torque developed
   c) Rotor copper losses
   d) Stator copper losses
   e) Stator iron losses
   f) Overall efficiency

4) A 440 V, 3 phase, 8 pole, 50 Hz, 40 kW, star connected three phase induction motor has the following parameters:
   - Stator resistance = 0.1 Ω, stator reactance = 0.4 Ω
   - Equivalent rotor resistance referred to stator = 0.15 Ω
   - Equivalent rotor reactance referred to stator = ~~0.15 Ω~~ 0.44 Ω

   The stator core loss is 1250 W while mechanical loss is 1000 W. It draws a no load current of ~~25 A~~ 20 A at 0.09 p.f. lagging, while running at a speed of 727.5 r.p.m., calculate:
   a) Input line current and p.f.
   b) Torque developed
   c) Output power
   d) Efficiency.......... Use approximate equivalent circuit.

sheet (4) [3Φ-IM]

5) A 4 pole, 3 phase, 50 Hz, slip ring type induction motor has a rotor resistance of 0.25 Ω per phase and rotor reactance of 2 Ω per phase at standstill condition. It is running at 1455 r.p.m. speed. Calculate the value of external resistance per phase required in the rotor circuit to reduce the speed by 17%. Assume load torque constant.

6) A 1000 V,50 Hz, 3 phase induction motor has star connected stator. The ratio of stator to rotor turns is 3.6. The standstill impedance of rotor per phase is 0.01+j0.2 Ω. Calculate:

a) Rotor current at start
b) Rotor p.f. at start
c) Rotor current at slip of 3%
d) External resistance per phase in the rotor circuit to limit starting rotor current to 200 A.

Best wishes
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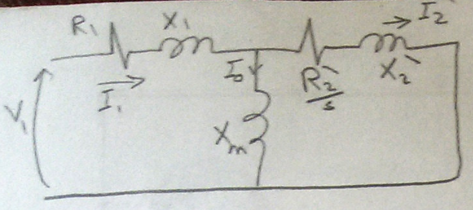
2013-2014



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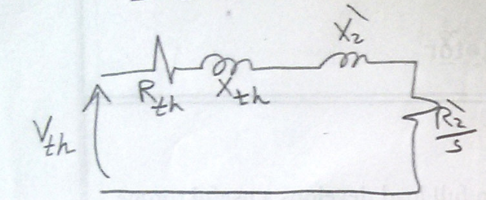


$$Z_{th} = (R_1 + jX_1) \,//\, jX_m$$

$$V_{th} = V_1 \frac{jX_m}{(R_1 + jX_1) + jX_m}$$

$$T_{dev} = max \Rightarrow \frac{R_2'}{s} = \left|(Z_{th} + jX_2')\right|$$

$$\Rightarrow S_{T_m} = \frac{R_2'}{\sqrt{R_{th}^2 + (X_{th} + X_2')^2}}$$

$$T_{dev} = \frac{3}{\omega_s} \cdot \frac{V_{th}^2 \; \frac{R_2}{s}}{\left(R_{th} + \frac{R_2'}{s}\right)^2 + (X_{th} + X_2')^2}$$

max. torque

max. power

$$P_{dev} = 3\, I_2'^2\, R_L'$$

$$\Rightarrow R_L' = \frac{1-s}{s} R_2'$$

$$P_{dev} = 3 R_L' \left[\frac{V_{th}^2}{(R_{eq} + R_L')^2 + X_{eq}^2}\right]$$

$$P_{dev} = max \Rightarrow \frac{\partial P_{dev}}{\partial R_L'} = 0$$

$$\frac{3V_{th}^2}{(R_{eq} + R_L')^2 + X_{eq}^2} + \frac{3R_L' V_{th}^2 \left(-2(R_{eq} + R_L')\right)}{\left[(R_{eq} + R_L')^2 + X_{eq}^2\right]^2} = 0$$

$$(R_{eq} + R_L')^2 + X_{eq}^2 - 2R_{eq}R_L' - 2R_L'^2 = 0$$

$$R_{eq}^2 + X_{eq}^2 - R_L'^2 = 0$$

$$R_L' = \sqrt{R_{eq}^2 + X_{eq}^2} = Z_{eq}$$

$$\frac{1-s}{s} R_2' = Z_{eq}$$

$$S_{P_m} = \frac{R_2'}{R_2' + Z_{eq}}$$

$$\Rightarrow Z_{eq} = \sqrt{R_{eq}^2 + X_{eq}^2}$$

$$R_{eq} = R_{th} + R_2$$

$$X_{eq} = X_{th} + X_2'$$

$$\frac{T}{T_{max}} = \frac{2 S S_m}{\left[S^2 + S_m^2 \right]}$$

Sheet (4) [3ϕ-IM]

$\boxed{1}$ 3ϕ-Y-IM, 4-pole, 50 HZ

$T_{L_{F.L}} = 300 \ N.m$, $f_r = 2 \ HZ$, $T_{rot} = 50 \ N.m$

Req:- s, P_o, $P_{cu_{rotor\,phase}}$, η_{rotor}, $[R_r$ if $I_r = 60 \ A]$

$$S = \frac{f_r}{f_s} = \frac{2}{50} = 0.04$$

$$N_m = (1-s) N_s = (1-s)\left(\frac{120 f_s}{P}\right) = 1440 \ rpm$$

$$P_o = T_L * \left(\frac{2\pi N_m}{60}\right) = 45.24 \ Kw$$

$$P_{dev} = (T_L + T_{rot}) * \left(\frac{2\pi N_m}{60}\right) = 52.78 \ Kw$$

$$P_{cu_r} = \frac{s}{1-s} P_{dev} = 2.199 \ Kw$$

or

$$P_{dev} = 3 I_2'^2 R_2' \left(\frac{1-s}{s}\right)$$

$$\therefore R_2' = 0.204 \ \Omega$$

$$\therefore P_{cu_{phase}} = I_2'^2 R_2' = 733.04 \ W$$

$$\therefore \eta_{rotor} = \frac{P_{dev}}{P_{gap}} * 100\%$$

$$= (1-s) * 100\% = 96\%$$

25 Kw, 4-pole, 50 Hz, 3ϕ-IM

$N_{m_{f.L}} = 1410$ rpm , $P_{rot} = 850$ W , $\left[P_{cu_s} = 1.7\, P_{cu_r} \text{ on Full Load} \right]$

Req:- P_{dev} , P_{cu_r} , R'_2 , η $\left[I'_2 = 65\,A \right]$

$$P_{dev} = P_{o_{f.L}} + P_{rot} = 25*10^3 + 850 = 25.85 \text{ Kw}$$

$$S = \frac{N_s - N_m}{N_s} = \frac{1500 - 1410}{1500} = 0.06$$

$$P_{cu_r} = \frac{S}{1-S} P_{dev} = 1.65 \text{ Kw}$$

$$\therefore P_{cu_s} = 1.7\, P_{cu_r} = 2.805 \text{ Kw}$$

$$\therefore P_{i/p} = P_{dev} + P_{cu_r} + P_{cu_s} = 30.305 \text{ Kw}$$

$$\therefore \eta = \frac{P_o}{P_{i/p}} * 100\% = 82.5\%$$

$$P_{cu_r} = 3\, I_2'^2\, R'_2$$

$$\therefore R'_2 = 0.1302\ \Omega$$

3ϕ, 50 Hz, 8-pole, 24 Kw, 415 v

$N_{m_{f.L}} = 720$ rpm , $I_{Line} = 57$ A , pf $= 0.707$ lag

$R_1 = 0.1\,\Omega$, $P_{rot} = 1000$ W

Req:- T_{sh} , T_{dev} , P_{cu_r} , P_{cu_s} , P_{iron} , η

$$T_{sh} = \frac{P_o}{\left(\frac{2\pi N_m}{60}\right)} = \frac{24\times 10^3}{\left[\frac{2\pi(720)}{60}\right]} = 318.31 \text{ N.m}$$

$$T_{dev} = \frac{P_o + P_{rot}}{\left(\frac{2\pi N_m}{60}\right)} = 331.573 \text{ N.m}$$

$$s = \frac{N_s - N_m}{N_s} = 0.04$$

$$P_{cu_r} = \frac{s}{1-s} P_{dev} = \frac{s}{1-s}(P_o + P_{rot}) = 1.042 \text{ Kw}$$

$$P_{cu_s} = 3\, I_{Line}^2\, R_1 = 974.7 \text{ W}$$

$$P_{in} = 3\, V_{ph}\, I_{Line}\, pf = 3\left(\frac{415}{\sqrt{3}}\right)(57)(0.707) = 28.97 \text{ Kw}$$

$$P_{iron} = P_{in} - P_{dev} - P_{cu_r} - P_{cu_s} = 1.95 \text{ Kw}$$

$$\eta = \frac{P_o}{P_{in}} * 100\% = 82.84\%$$

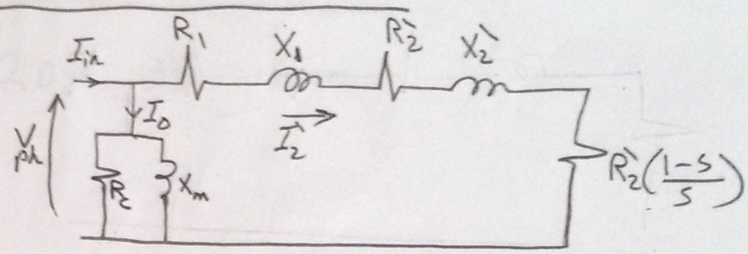
3ϕ, 440v, 50 Hz, 8-pole, 40 Kw, Y - IM

$R_1 = 0.1\ \Omega$, $X_1 = 0.4\ \Omega$, $R_2' = 0.15\ \Omega$, $X_2' = 0.44\ \Omega$

$P_{core} = 1250\ W$, $P_{rot} = 1000\ W$

$I_{n.L} = 20\ A$, $pf_{n.L} = 0.09$ lag , $N_{m\ operating} = 727.5$ rpm

Req:- I_{in} , pf_{in} , T_{dev} , P_o , η [use approximate ckt]



$I_o = I_{n.L} \angle -\cos^{-1} pf_{n.L}$

$I_o = 20 \angle -84.84$ A

$I_2' = \dfrac{V_{ph}}{\left(R_1 + \frac{R_2'}{s}\right) + j(X_1 + X_2')} \Rightarrow s = \dfrac{N_s - N_m}{N_s} = 0.03$

$\therefore\ I_2' = 49.1484 \angle -9.353$ A

$\therefore\ I_{in} = I_2' + I_o = 57.5172 \angle -29.024$ A

$\therefore\ pf_{in} = \cos(29.024) = 0.8744$ lag

$P_{dev} = 3 I_2'^2 R_2' \left(\frac{1-s}{s}\right) = 35.1465$ Kw

$T_{dev} = \dfrac{P_{dev}}{\left(\frac{2\pi N_m}{60}\right)} = 461.34$ N.m

$P_o = P_{dev} - P_{rot} = 34.1465$ Kw

$P_{in} = 3 V_{ph} I_{in} pf_{in} = 38.33$ Kw

$\eta = \dfrac{P_o}{P_{in}} * 100\% = 89.1\ \%$

3ϕ, 50 Hz, 4-pole, slip ring IM

$R_2' = 0.25\,\Omega$ $X_2' = 2\,\Omega$ at (stand still)

$N_m = 1455$ rpm

Req:- R_{add} in rotor circuit to reduce speed by 17%.

[assume Load torque constant]

$N_m' = 0.83\, N_m = 1207.65$ rpm

$$S = \frac{N_s - N_m}{N_s} = 0.03$$

$$S' = \frac{N_s - N_m'}{N_s} = 0.1949$$

$$T_{dev} = \frac{3\, I_2'^2 R_2' \left(\frac{1-s}{s}\right)}{\omega_m} = \frac{3\, I_2'^2 (R_2'/s)}{\omega_s}$$

$$I_2' = \frac{E_2}{\sqrt{\left(\frac{R_2'}{s}\right)^2 + X_2'^2}}$$

$$\therefore T_{dev} \propto \frac{\left(\frac{R_2'}{s}\right)}{\left[\left(\frac{R_2'}{s}\right)^2 + X_2'^2\right]}$$

$$\because T = \text{Const} \implies \therefore \frac{\left(\frac{R_2'}{s}\right)}{\left[\left(\frac{R_2'}{s}\right)^2 + X_2'^2\right]} = \frac{\left(\frac{R_{2new}'}{s'}\right)}{\left[\left(\frac{R_{2new}'}{s'}\right)^2 + X_2'^2\right]}$$

$$\therefore R_{2new}'^2 - 1.71772\, R_{2new} + 0.151944 = 0$$

$$\therefore R_{2new}' = 1.6242\,\Omega$$

$$\therefore R_{add} = R_{2new}' - R_2' = 1.3742\,\Omega$$

3ϕ, 1000v, 50 Hz, Y-IM

$\frac{T_{ph_s}}{T_{ph_r}} = 3.6$, $R_r = 0.01\,\Omega$, $X_r = 0.2\,\Omega$

Req:- $I_{r_{st}}$, $pf_{r_{st}}$, $I_{r_{(s=0.03)}}$, R_{add} to [Limit $I_{r_{st}}$ to 200 A]

$$E_r = E_s * \frac{T_{ph_r}}{T_{ph_s}} = \frac{\left(\frac{1000}{\sqrt{3}}\right)}{3.6} = 160.375 \text{ v}$$

$$I_r = \frac{E_r}{\sqrt{\left(\frac{R_r}{s}\right)^2 + X_r^2}} \qquad pf_r = \frac{(R_r/s)}{\sqrt{\left(\frac{R_r}{s}\right)^2 + X_r^2}}$$

* at [starting (s=1)]

$I_r = 800.8745$ A

$pf_r = 0.05$ lag

* at [s = 0.03]

$I_r = 412.56$ A

$$\frac{I_{r_{st_{new}}}}{I_{r_{st}}} = \frac{\sqrt{R_r^2 + X_r^2}}{\sqrt{(R_r + R_{add})^2 + X_r^2}}$$

$\therefore R_{add} = 0.7665\,\Omega$